A Lightweight Tandem Bike Frame
Finite Element Analysis Report

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Contents

1. Introduction

The brief for this project is to design a lightweight tandem bicycle frame and assess its functionality using finite element (FE) analysis. To avoid user discomfort due to vibrations, the natural frequency of the frame should be greater than 30 Hz, and to ensure it has an expected lifespan of 10 years, it should withstand one million loading cycles.

Two frame geometries (A and B) were modelled in Solidworks and each was simulated with an Aluminium and a Titanium alloy, resulting in four frame iterations. This report finds the natural frequency and fatigue lives of each iteration using FEA.

2. Methods

2.1 Designs

The frame geometry is the 'direct-internal' shape [1] scaled to fit the dimensions outlined in the brief: a wheel diameter of 66 cm, wheel spacing of 1.5-2m and seat height of 80cm. The dimensions of frame A, including diameters and thicknesses of the structural members, are shown in Figure 1.

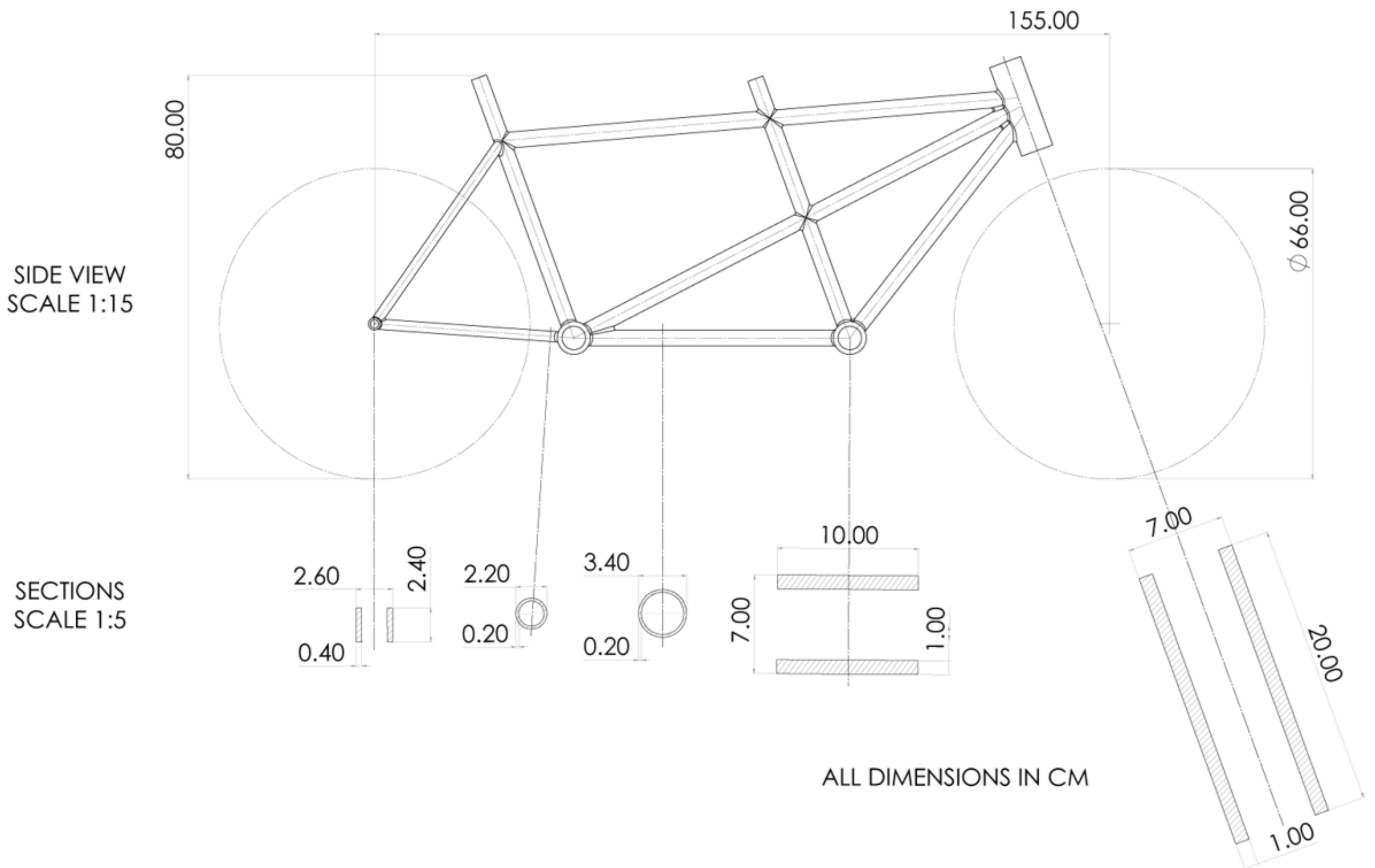

Figure 1 - A dimensioned side view of the bike frame design.

Frame B was designed as a lighter-weight alternative; its 34mm tubes have a wall thickness of 1 mm rather than 2 mm. Their outer diameter and all other frame dimensions are unchanged.

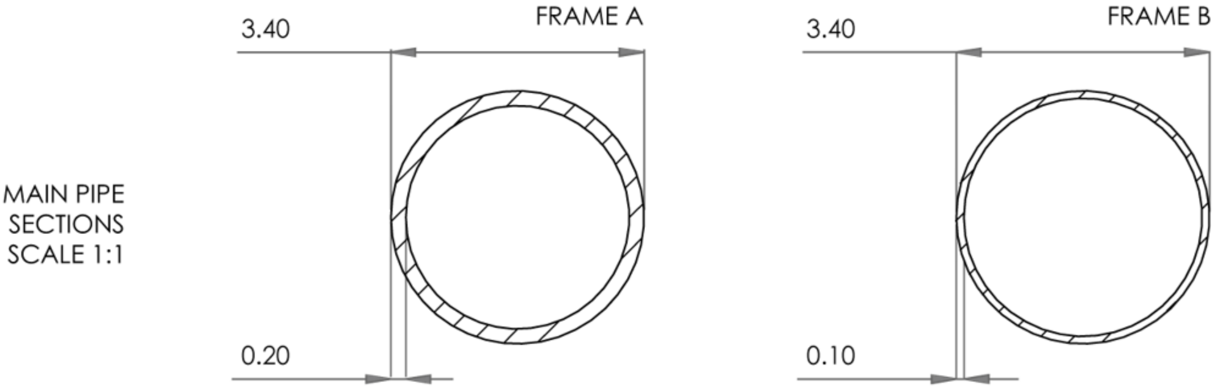

Figure 2 - The difference in tubing between frame A and frame B.

2.2 Materials and Geometry Assumptions

The materials used are Aluminium 7075-T6, as given in the brief, and Titanium 6Al-4V, which is an alloy used in high-end bike frames [2].

Table 1 - The material properties of the Aluminium and Titanium alloys used.

Material	Tensile Strength (MPa)	Young's Modulus (MPa)	Density (kg/m^3)	Poisson's Ratio
Al 7075-T6	570	72.0	2810	0.33
Ti 6Al-4V	1050	104.8	4429	0.31

The frame tubes are modelled as having perfectly circular cross-sections with uniform thickness walls along their lengths. The welds are modelled with 5mm fillets. The Aluminium and Titanium are assumed to be isotropic and homogenous, meaning that they have uniform properties in all directions and in every position in the frame.

In reality, there will be geometric imperfections in the manufactured product, so the welds and tubing will not be perfectly uniform. There will additionally be imperfections in the material, especially in the weld areas: the joining process creates heat-affected and fusion zones which have altered material properties and residual stresses.

2.3 Loading and Boundary Conditions

The simulations model two 150 kg cyclists pedalling in synchronisation. Their masses are modelled as forces of 1471.5 N acting perpendicular to the upper face of each seat tube. Each cyclist's pedalling is modelled as an oscillating force of 0 to 750 N acting vertically downwards on the inner face of the crank shell. The pedals are not modelled and are assumed to be rigid, so these forces are modelled as acting 200 mm in front of, and 100 mm sideways from the centre of each crank shell. The stress created in the frame due to its weight will be minimal, so gravitational force is not included.

The fork shell is fixed, meaning it has translational degrees of freedom. The rear wheel bearings are hinged, meaning they can rotate along the Z-axis (parallel to the ground), to represent the rotation of the rear wheel.

These conditions are approximations of the real loading conditions a tandem frame will experience, which will be variable and difficult to predict and model.

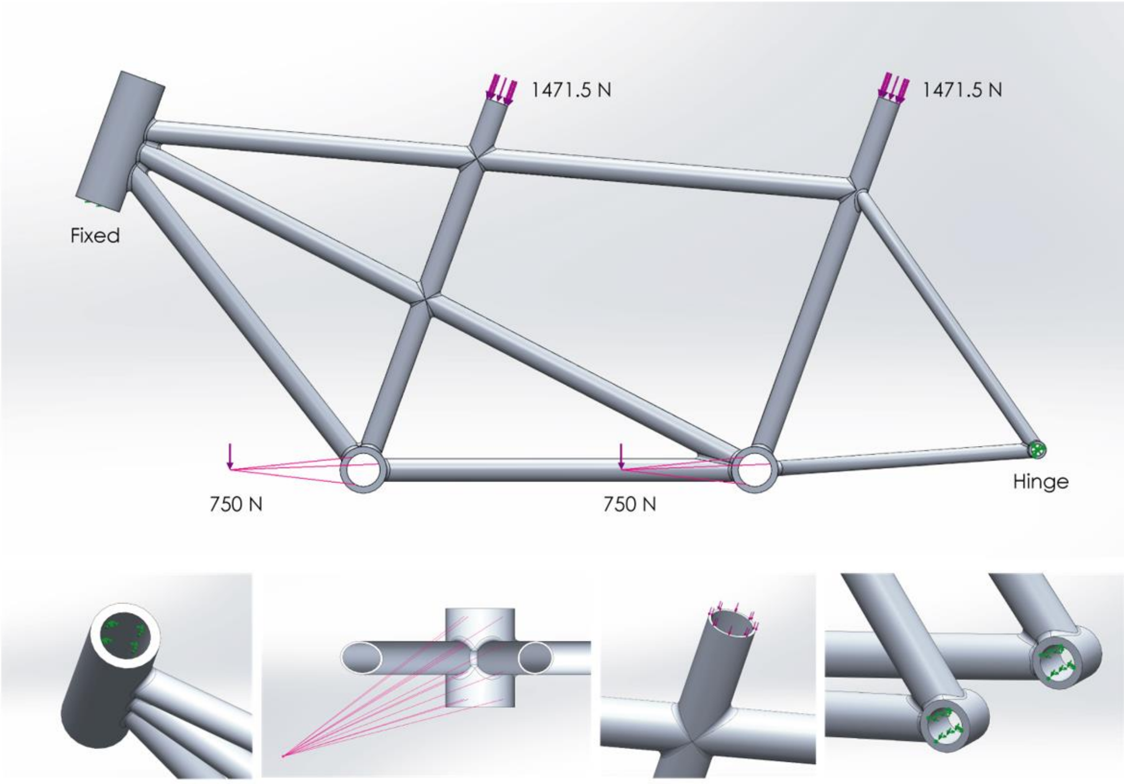

Figure 3 - The loads and boundary conditions applied in the different studies.

2.4 Linearity

Aluminium and Titanium have linear stress-strain relationships below their elastic limit, as defined by their Young's moduli. The stress on the frames is expected to remain below this limit and their deformations are expected to be small. Therefore, the problem is assumed to have material and geometric linearity. Since the seats and pedals are not modelled and are assumed to be rigid, the force contact areas are constant. This means the problem also has contact linearity, so this report uses linear analysis, which uses constant stiffness matrices.

2.5 Simulations

Frames A and B were each simulated using Aluminium and Titanium. Each of the following studies was run for all four iterations.

Static

Two static studies were run to determine the maximum stress in the frame. The left study models both cyclists applying the maximum 750N to the left of the crank shell, as described under loading condition, and the right study models them applying the same force on the right. The cyclists' masses and the boundary conditions on the fork and rear bearings are also included. The left and right studies are mirror images, and the frame is symmetrical, so the maximum stress is expected to be the same for both studies.

Fatigue

To determine the fatigue life of the frame the FE method uses an S-N curve which demonstrates the relationship between the alternating stress (S) and number of stress cycles (N). The Al 7075–T6 S-N curve is provided in the Solidworks material library and the Ti 6A-4V curve was obtained from the Solidworks extended material library.

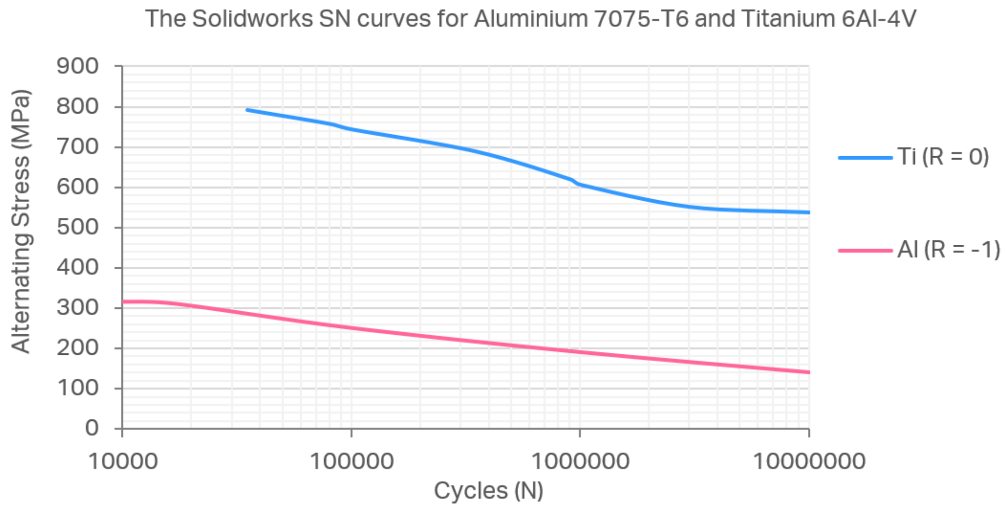

Figure 4 - The material SN curves showing their expected life for different alternating stress values.

The Al curve (Figure 4) shows the S-N relationship for a stress ratio ($R = \sigma_{min}/\sigma_{max}$) of -1, which occurs when the stress oscillates between positive and negative values of the same amplitude. However, in this situation, the pedalling forces oscillate between 0 and 750 N, which is a stress ratio of 0. Therefore, the Gerber method, which is suitable for ductile materials, is used to obtain the corrected alternating stress values using the alloy's mean stress and ultimate strength [3]. The Solidworks Ti curve used is provided for R = 0 so is not corrected.

The fatigue study uses the loads from the left and right static simulations as fatigue events. Although the seats do not experience consistent cyclic loading like the pedals, the loading on the seats is included because it does vary during use: cyclists transfer weight off the seat during pedalling, stand up to move over or around obstacles, and step off the seat at stops.

This report uses the 'Find Cycle Peaks' loading type to combine these loads in the manner that creates the greatest stress variation [4]. This is a worst-case loading scenario and is likely to simulate a higher stress variation than would occur in regular use when the pedals are loaded consecutively and the force on the seat is largely constant.

Using static studies models the loading as quasi-static, meaning that the inertial effects of the forces are ignored [3]. This is acceptable because the frequency of the loading (pedalling at 1-2 Hz) is expected to be significantly lower than the natural frequencies of the frame (>30 Hz).

Frequency

The frequency study determines the natural frequencies of the frame and their corresponding mode shapes. The fixtures described above are retained because they will constrain the frame relative to the wheels and ground in regular use. The loading on the seats and pedals will vary during use, so the frequency study was run with and without these forces for Frame A in Aluminium. The difference in results was minimal (0.9 Hz / 1.2%), so loading was not included in the subsequent frequency simulations.

$$\omega = \sqrt{k/m} \qquad (1)$$

Decreasing mass (m) or increasing stiffness (k) increases the natural frequencies of an object, as shown by Equation 1, which finds the fundamental-frequency of an undamped, one-degree of freedom system. Therefore, natural frequencies are impacted by both material properties and object geometry.

3. Results

3.1 Mesh Refinement

The models are meshed using curvature-based mesh. This automatically uses smaller elements in high-curvature areas, so the frame can be meshed with a larger global mesh size [5]. Initially, large global mesh sizes of 10mm and 5mm were used to identify the areas of stress concentration. (The meshing of frame B failed at 10mm, so a global mesh size of 5mm was used; this is probably because the thinner tubing creates smaller details around the edges, which requires a finer mesh.)

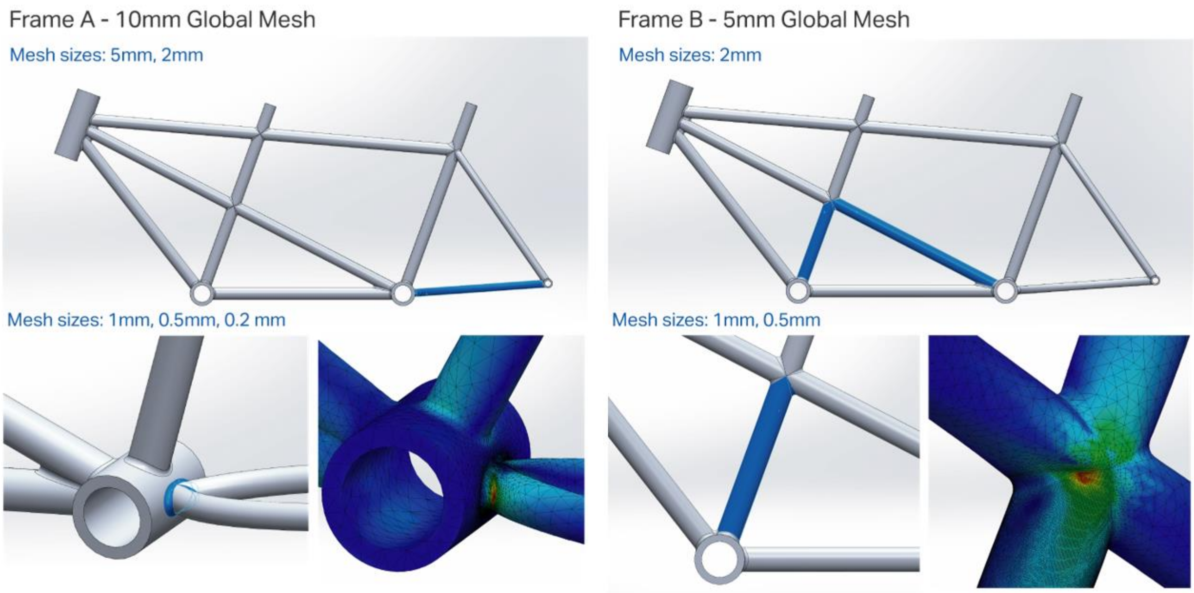

Figure 5 - The mesh refinement stages and locations for the static studies.

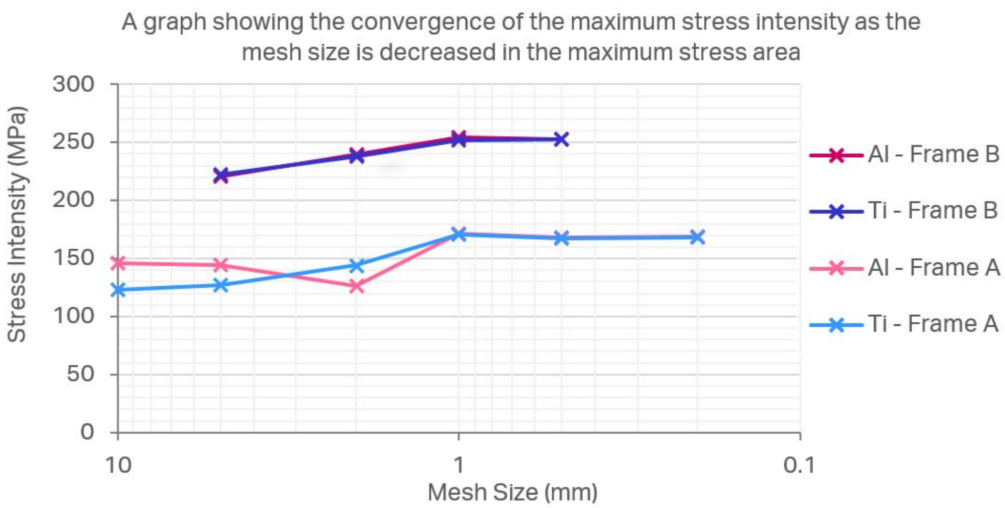

Figure 6 - The mesh sizes and static simulation stress results obtained during mesh refinement.

For the static studies, the mesh size in these high-stress areas was systematically reduced until the maximum stress value converged. These refinement locations and mesh sizes are shown in Figure 5. As previously justified, the left and right study results are expected to be the same, so this mesh refinement was performed for the left pedalling force only, and the selected refined mesh sizes were mirrored for the right-hand study.

The static mesh refinement results are shown in Figure 6. Computational limitations prevented smaller mesh sizes being tested. However, this is not a major issue, because the consecutive results for 1 and 0.5 mm converge to within 1%. Therefore, 0.5mm mesh is used in the high-stress areas identified in Figure 5.

For the frequency study, the global mesh size was reduced, because the displacements due to frequency occur across the frame and in varying locations dependant on the mode. The smallest mesh size that could be tested was 5mm. The 10mm and 5mm results for frame A converged to within 1% of each other and, as previously mentioned, frame B could not be meshed at 10mm. Therefore, a global mesh size of 5mm is used.

3.2 Mesh Quality

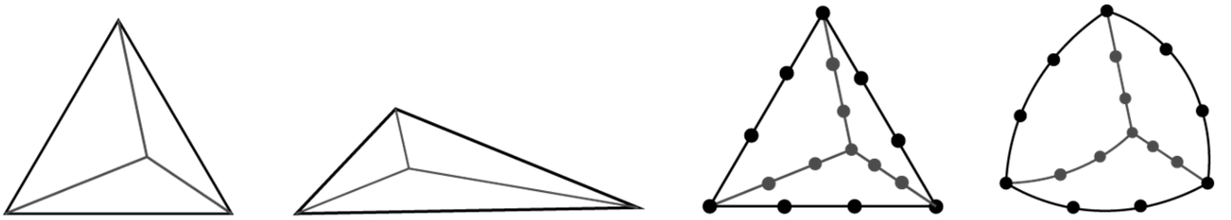

Figure 7 - An illustration of the aspect ratio and Jacobian ratio of tetrahedral elements.

The aspect ratio of an element is the ratio of the length of its longest side to its shortest normal between a vertex and opposite face, normalised to give an of ideal value of one for a regular tetrahedral [6].

The Jacobian ratio is a measure of the displacement of points on the element's edges relative to an ideal element (regular tetrahedral). This report checks this ratio using 16 points, as shown in Figure 7. If the edges of an element intersect, the Jacobian is negative, and meshing returns an error. Solidworks deems a Jacobian of less than 40 as acceptable [6].

Table 2 - Mesh details demonstrating the mesh quality of the refined meshes used for the simulations.

Frame	Study	Aspect Ratio			Distorted Elements
		% <3	% >10	Max	
A	Static/Fatigue	61.1	0.738	26.8	0
A	Frequency	94.4	<0.001	10.2	0
B	Static/Fatigue	31.3	1.79	26.5	0
B	Frequency	48.4	0.021	13.6	0

Optimising the aspect and Jacobian ratios to be near one for all elements is important to improve the mesh quality and resulting model accuracy. As shown in Table 2, the refined meshes for the different studies are of varying qualities. For the static studies, the elements with high aspect ratios are not found near the areas with the highest stress concentration, where the results are most important. Since there are no distorted elements, and the simulation results appear to converge with the mesh sizes used, these meshes are deemed to be of acceptable quality.

3.3 Study Results

Aluminium and Titanium are ductile materials, so the von-Mises or Tresca criterion can be used to determine whether the material fails. This report does not use a safety factor, so it uses the Tresca criterion (stress intensity), because it gives slightly more conservative results. The results are presented in Table 3.

Table 3 - The results of the static, fatigue and frequency studies.

	Design	Material	Mass (kg)	Max Stress Intensity (MPa)	Max Damage % (1 million cycles)	Fundamental Frequency (Hz)	Pass /Fail
1	A	Al 7075-T6	5.11	167.8	NA	71.67	Pass
2	A	Ti 6Al-4V	8.06	167.2	NA	68.96	Pass
3	B	Al 7075-T6	3.98	252.5	100.2%	61.36	Fail
4	B	Ti 6Al-4V	6.27	252.4	NA	59.05	Pass

Static

The maximum stress intensity on all four frames is below the tensile strength of the material, so they are not expected to fail under static load.

Fatigue

The fatigue studies return the damage percentage across the frame. This is the percentage of the frame's life before failure that has been used by the given number of stress cycles. A result greater than 100% shows the frame has failed in that location.

The alternating stresses in frame A and the Titanium frame B are below the asymptote of the S-N curves for their materials. This means the stresses are below the fatigue limit and, theoretically, the frame can sustain an infinite number of stress cycles. On Solidworks, this returns a life of 40 or 10 million cycles, because these are the highest data points on the Al and Ti SN curves used.

The Aluminium frame B experiences stresses above its fatigue limit. It is expected to fail after 998,300 cycles, so does not pass the one million cycle criteria.

Frequency

All the frames have natural frequencies which are significantly higher than the minimum acceptable value of 30 Hz, so they will not cause discomfort due to matching natural frequencies of the body.

Since the frequency results from frame A were significantly greater than 30 Hz, frame B was designed to decrease weight, rather than to improve the natural frequency. Decreasing the pipe thickness reduces the frame stiffness, which, as discussed in the methods section, will decrease its natural frequency. Therefore, frame B has lower natural frequencies than frame A (while still passing the brief criteria).

3.4 Sanity checks

Several features indicate the model and results are approximately accurate:

- The masses and centre-of-masses of the frames are logical values.
- The maximum stress intensities are located on the surface, where maximum bending stresses are found. The highest fatigue-damage percentages are in the same locations.
- The maximum stress intensities are almost identical for both materials for each frame geometry. This is logical as stress is a function of geometry and load, not material properties.
- The fatigue study results are logical when compared to the material SN curves.
- The stress intensity and damage in frame B are higher than in frame A, since it has thinner tube walls.
- The displacements of all the loaded frames in the static studies are small (in the order of mm).
- Frame B has lower natural frequencies than frame A (as expected using Equation 1).

4. Discussion

4.1 FEA Limitations

The sources of error in FEA can be grouped into three categories: modelling, discretisation and numerical [3].

Modelling error is due to differences between the real-life problem and its FE model. As previously mentioned, the geometry, material properties and loading of a physical bike frame will be variable and will not be identical to the idealised scenario simulated in this report.

Discretisation error is caused by the approximation of the model as finite elements. This has been minimised by refining the mesh in key areas so converging results are found and by ensuring that the mesh quality is suitably high.

Numerical error is a result of finite precision being used to store simulation data. This leads to the accumulation of discrepancies like rounding errors. It can be reduced by increasing the precision of the variables. However, this increases the computational power and time needed.

4.2 Conclusions

This report finds that three of the four iterations meet the criteria of having an effective life greater than 10 years, and a fundamental frequency above 30 Hz. Of these, design A in Aluminium is the lightest, weighing 5.11 kg, so this is chosen as the best design. It could further be optimised by testing additional geometries and alloys.

The FE models presented in this report are approximations of real tandem frames and their use. As discussed in the methods and limitations sections, they do not perfectly represent the variations in material, geometry and loading in real products and use, due to the way the problem is modelled, discretised, and numerically solved.

This report is likely to overestimate the loads the frame would experience: it models 150kg cyclists and combines the fatigue study loads in the manner that creates the maximum alternating stress. The results also use the conservative Tresca failure criterion. Since the sanity checks additionally indicate that the results are approximately accurate, this report predicts that the three frames which pass the criteria are extremely unlikely to fail in regular use.

However, despite this, the potential errors introduced when using FE method mean these frames and any other product simulated with FEA should be physically tested with real loading before wide-scale production and use.

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